INVESTMENT TIMING AND CAPACITY CHOICE FOR SMALL-SCALE WIND POWER UNDER UNCERTAINTY

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ABSTRACT

This paper presents a method for evaluation of investments in small-scale wind power under uncertainty. It is assumed that the price of electricity is uncertain and that an owner of a property with wind resources has a deferrable opportunity to invest in one wind power turbine within a capacity range. The model evaluates investment in a set of projects with different capacity. It is assumed that the owner substitutes own electricity load with electricity from the wind mill and sells excess electricity back to the grid on an hourly basis. The problem for the owner is to find the price levels at which it is optimal to invest, and in which capacity to invest. The results suggests it is optimal to wait for significantly higher prices than the net present value break-even. Optimal scale and timing depend on the expected price growth rate and the uncertainty in the future prices.

KEYWORDS

Renewable Generation, Distributed Generation, Risk Analysis, Project Evaluation, Electricity Markets, Investment Appraisal, Real Options

INTRODUCTION

In the transition to a renewable power system both central and distributed generation is needed. Small-scale wind turbines are among the emerging technologies with a potential to contribute to growth in the renewable share of power generation. The American Wind Energy Association (AWEA) has estimated that small wind turbines could contribute to 3 percent of U.S. electrical consumption within 2020 [1].

This paper analyzes the choice of capacity in small-scale wind power generation, as well as the choice of time (or price level) to invest in a liberalized market structure. The focus is on wind power generation that serves a customer on-site and exports excess electricity to the grid. Our approach is developed for an investor that can choose among certain given capacities within a capacity range. Investment in the different projects is assumed to be mutually exclusive.

Power production from wind power has stochastic inflows, and production profiles cannot be matched to load profiles, so to fully meet demand is impossible because sometimes there is no wind. We assume the investor is net metered hourly. The part of the production that covers the customer's own demand has a higher value to the investor, because the price of electricity bought on the grid is higher than the price of electricity sold back to the grid, due to grid tariffs, supplier mark-ups and taxes. Under such circumstances the marginal benefit/value as a function of capacity is declining because a larger share of the production is valued at the lower export price. At the same time the investment cost per kW is reduced with increased capacity.

A possible approach for the optimal sizing of the wind mill is to simply choose the capacity that maximizes the net present value. Note that by net present value we mean the present value of energy savings and exports less investment costs. Maximizing net present value with respect to capacity is the correct way to choose capacity, but this operation is somewhat complicated by the presence of uncertainty. We assume the wholesale price of electricity is uncertain, that the owner of the property has an opportunity to delay investment perpetually, and that the investment is irreversible. In this situation, there is a threshold level of electricity prices at which the marginal benefit of waiting for better information about electricity prices equals the marginal net present value of the investment. I.e. there is a threshold level of price that governs the optimal time to invest.

This contrasts with deterministic net present value analysis, where the optimal time to invest depends on the growth rate of prices or benefits, and where the value of waiting for better information is zero. Owners should invest in power production when the value of the investment opportunity (option) equals the present value of future benefits less investment costs. However, the optimal capacity varies with price. Therefore the project must also be the one with the maximum net present value at the investment threshold. In addition, the net present value must be higher here than the value of the investment opportunity in any of the larger projects.

The analysis is further complicated because an optimal investment under uncertainty will never occur at a kink in the net present value function. In our model there will be such a kink at the price where two projects of different scale have equal net present value. Around this indifference point it is optimal to wait for new information about the price development to decide which project is the best.

We develop a model to support and analyze the decision to invest in this situation. The model is illustrated using a case of investment in distributed wind power in the Nordic power market, based on one year of historical hourly wind speed and demand data for an individual customer.

In our example the recommended decision is to postpone the investment until the price is far higher than the net present value break-even price. Depending on growth rate and uncertainty in future prices, there exists a waiting region for prices, until investment in a first project is optimal. If the project is not the largest there will exist one or more new waiting regions around the price where two projects have the same net present value. The end of the waiting period marks the beginning of a new investment region. The investment threshold for the largest project is a trigger level where investment is optimal for all higher prices.

Higher uncertainty increases the investment price regions and the optimal size of the turbine. Increased expected growth increases both the profitability and the investment opportunity but reduces the price where it is optimal to invest in our example. However the optimal scale also increases as a function of increased expected price growth.

THE LONG-TERM DYNAMICS OF THE ELECTRICITY PRICE

Mean reversion is a well-known feature of prices in commodity markets, where the price has a tendency to revert to a long term average. Lucia and Schwartz [2] have studied the prices in the Nordic electricity market using one- and two-factor models. In the one factor model the price is assumed to follow a mean reverting Ornstein-Uhlenbeck process. In the two factor models the short term variations in the price were assumed to follow a similar process and the long term variations were assumed to follow arithmetic or geometric Brownian motion. The two factor models had the better fit to the data. However, Smith and Schwartz [3] argue that when considering long term investments the long term factor is the decisive factor. Similarly Pindyck [4] claims that when considering long-term commodity related investments, a geometric Brownian motion description of the price will not lead to large errors. Although using a geometric Brownian motion to model price dynamics ignores short term mean reversion, a wind mill must be regarded as a long term investment where the short term mean reversion barely influences values and investment decisions. Motivated by this, and due to the simple solutions obtainable for such a process, we assume the long term electricity prices follow a geometric Brownian motion, where the change in price over a small time interval is written as

$$dS = \alpha S dt + \sigma S dz \tag{1}$$

where α is the annual risk-adjusted growth rate and σ is the annual volatility. The last term dz is a standard Wiener process. The parameters of (1) are estimated from forward contracts with a long time to maturity. Thus (1) represents the *risk-adjusted* long term price dynamics. See for example [5] for a discussion about price processes.

We are using annual cash flow estimates in which spot prices vary each hour over a year, hence seasonal variations do not have to take into account in the price model. With the above price description the risk-adjusted expected future price is given as

$$S(t) = Se^{\alpha t} \tag{2}$$

where S, the current price, is the spot price adjusted for short-term deviations.

THE VALUE OF THE WINDMILL

For an investment analysis of a windmill we need to find the relation between the annual value of the wind power plant and future expected annual prices of electricity. The present value of the windmill is the sum of the present value of the reduced purchase of electricity and the value of the electricity sold to the grid less the operation and maintenance costs.

We have assumed the customer is net metered hourly; therefore the value of the windmill is dependent on the hourly wind power production, the hourly demand and the hourly spot price. These data have a complicated statistical nature. All three variables have seasonal and daily variation patterns, and are correlated through the influence of varying weather. A statistical time series analysis that takes into account the correlation between the variables is a huge task and will need long historical time series for all variables. In addition such time series are very often not available. Therefore, and in accordance with the discussion in [6], we find the annual cash flows in our model from the available historical hourly data.

In the model historical wind speeds and loads are considered representative for the future but the electricity price can change. Further, in the investment analysis we will use the annual average wind power production, load and electricity price. Therefore we collect all the information about the correlation between the price, load and wind power production in two factors that adjusts the average wholesale price of electricity. A factor, k_{id}, to adjust the average wholesale price for the part of production that substitutes own demand and an equivalent factor k_{ie} for electricity exports, where the index, i, represents the different windmills with different capacity. Now the product of k_{id} and S is the average wholesale price for the part of production that substitutes own demand, and the product

of k_{ie} and S the average wholesale price for exports. To find the factors we must first transform the hourly wind speed data to hourly power production. Then we have power production, load and price for each hour and finding the two factors is straightforward.

Under these circumstances the average electricity price received, when substituting own demand, is given as

$$P_{id}(t) = k_{id}S(t)v + v(s+g+e) \tag{3}$$

where k_{id} is the adjustment factor for the average wholesale price, S is the annual average wholesale price, v is the value added tax, s is the supplier mark-up, g is the grid tariff and e is an electricity tax. The electricity price relevant when exporting to the grid is assumed to be

$$P_{ie}(t) = k_{ie}S(t) - s \tag{4}$$

where, k_{ie}, is the adjustment factor for the average wholesale price and the supplier mark-up, s, is assumed to be the same as when electricity is imported.

The annual income from owning wind mill, i, can thus be calculated as

$$x_i(t) = Q_{id}P_{id}(t) + Q_{ie}P_{ie} - C_{iO\&M} = a_i + b_iS(t) \tag{5}$$

where Q_{id} is the expected annual wind power production that reduces demand, Q_{ie} the corresponding production that is exported to the grid, $C_{iO\&M}$ is operation and maintenance cost and r is the risk-free discount rate. The present value of an investment in a windmill is

$$\begin{aligned} v_i &= \int_0^T (a_i + b_iSe^{\alpha t})e^{-rt}dt \\ &= \frac{a_i}{r}(1-e^{-rT}) + \frac{b_i}{r-\alpha}(1-e^{(\alpha-r)T})S \\ &= \theta_i + \gamma_i S \end{aligned} \tag{6}$$

where T is the expected lifetime of the windmill. The expected net present value of the project is simply, $v_i - I_i$. Someone contemplating to invest now will invest in the project with the highest net present value at this time.

THE INVESTMENT MODEL

Since we consider postponing the investment we will in the rest of the paper also take into account the value after a project dies. After a project a dies one will usually have the option to invest in any of the projects considered. However, since one often will not build a smaller project (because the opportunity to invest in a larger project is more worth than investing in the smaller), and for analytic simplicity, we assume the only investment opportunity left after a project dies is to invest in the largest project. We assume we have N projects to choose between. We further denote the investment possibility in the largest project, if analyzed alone, as $F_N(S)$, and the investment price threshold for the largest project p_N. The value functions, which represent the expected value of the sum of all further projects, will have two branches. At the first branch from S equal zero to $S = e^{-\alpha T}p_N$ the value function is the sum of the present value of the project and the present value of the expected value of the option to reinvest in the large project

$$V_i = \theta_i + \gamma_i S + \varepsilon(e^{-rT}F_N(S(T))) \tag{7}$$

from the price $S = e^{-\alpha T}p_N$ reinvestment in the large project is expected to be immediate after the project dies and the value function is given as

$$V = \theta_i + \gamma_i S + e^{-rT}\left(\frac{a_N}{r} + \frac{b_N}{(r-\alpha)}S(T)\right) - \frac{I_N}{e^{rT}-1} \tag{8}$$

The two branches of the value functions meet tangentially at $S = e^{-\alpha T}p_N$.

First we analyze the N projects individually. We assume the investment opportunity in the projects, $F_i(S)$, yields no cash flows up to the time the investment is undertaken. The only return from holding it is the capital appreciation. The Bellman equation for investment is [5]

$$rF_i(S)dt = \varepsilon^*(dF_i) \tag{9}$$

where ε^* denotes risk-adjusted expected value. Expanding $F_i(S)$ using Ito`s lemma [5] and taking the risk-neutral expectations gives

$$\frac{1}{2}\sigma^2S^2F_i`` + \alpha SF_i` - rF_i = 0 \tag{10}$$

A solution of the differential equation is $F_i(S) = A_iS^{\beta_1}$, where A_i is a constant to be determined, and β_1 is given by the positive solution of the quadratic equation resulting from substituting the solution into the differential equation. To find the constant, A_i, and the optimal investment thresholds, p_i, we need two boundary conditions for each project [5]. The first

$$F_i(p_i) = V_i(p_i) - I_i \tag{11}$$

states that optimal investment timing is when the value of the option to invest equals the net present value of the project. The second says that the derivative of the option value at the optimal threshold must equal the derivative of the project:

$$F_i'(p_i) = V_i'(p_i) \tag{12}$$

For prices above the investment threshold, the investment opportunity is worth the same as the net present value of the project. We must work backwards by first finding A_N and p_N on the second value branch. Following [7] it is now optimal to wait until

$$\begin{aligned} &p^* = \min_i p_i \\ &\text{s.t. } F_i(p_i) = \max_j F_j(p_i), i = 1..N, j = 1..N \end{aligned} \tag{13}$$

This can be interpreted as wait until the lowest p_i where the option to invest in that project is worth more than the option to invest in any of the other projects. If the lowest threshold price satisfying (13) is $p^* = p_N$ the solution is completed, and it is a trigger price where investment is optimal in the largest project for all higher prices, and waiting is optimal for all lower prices. However, if $p^* \le p_N$ there is an intermediate solution where a smaller project is optimal for some prices and one or more larger projects are optimal for higher prices. Investment in the

project, i, that is optimal for the lowest prices will then be optimal in a region from $p_{i,1}$ to $p_{i,2}$ where $p_{i,1} = p*$

Décamps et al [8] have showed that it is not optimal to invest at the point where the upper net present value function exhibits a kink, because there is uncertainty about which project is most profitable. When two projects of the same size have the same net present value such kink will exist, we will refer to this points as the indifference points. There will hence be a waiting region from $p_{i,2}$ until $p_{i+1,1}$. Investment in the largest project will be optimal for all values over $p_{N,1}$. Now the solution consists of a set of one or more investment regions, $P_i = [p_{i,1}, p_{i,2}]$.

The investment opportunity, F_m, around each indifference point, m, must satisfy the following boundary equations [8]

$$\begin{aligned} F_m(p_{i,2}) &= V_i(p_{i,2}) - I_i \\ F_m'(p_{i,2}) &= V_i'(p_{i,2}) \\ F_m(p_{i+1,1}) &= V_{i+1}(p_{i+1,1}) - I_{i+1} \\ F_m'(p_{i+1,1}) &= V_{i+1}'(p_{i+1,1}) \end{aligned} \tag{14}$$

The solution to the differential equation that satisfies the boundary equations is

$$F_m(S) = C_m S^{\beta_1} + D_m S^{\beta_2} \tag{15}$$

where β_2 is the negative solution to the quadratic equation. The four unknown parameters can be found from the four equations. There exists no analytic solution hence the solution must be found using numerical methods.

RESULTS

In this section we present the results based on our example with a one year record of hourly price, demand and wind data. The customer has a 42 kW maximum power need and annual consumption of 137.8 MWh, which can be representative for a small farm. The wind data has an average wind speed of 6.9 m/s. Additional approximate data are provided in Table 1. The electricity price parameters are considered representative of long-term forward contracts traded at Nord Pool, the Nordic Power Exchange. The remaining parameters have values that are typical for a Norwegian setting.

Table 1: Approximate values for the parameters used in the model.

Parameter	Value	
α	0.01	[]
σ	0.075	[]
r	0.05	[]
v	1.24	[]
g	35	[$/MWh]
e	14	[$/MWh]
s	3	[$/MWh]
$C_{O\&M}$	2	[% of I p.a.]
T	25	[y]

Table 2 shows the data and modeling results for the five wind turbines we consider to invest in. Turbines with a higher capacity, C, are assumed to have a lower investment cost per kW output. The information from the modeled values of Q_d and Q_e shows that for small turbines most of the production substitutes own demand and for larger turbines most of the production is for exports. The values of k_d and k_e show that there is a significant variation in the average price received for substituting own demand and exports due to seasonal patterns in demand, production and the price.

Table 2: Generator data and modeling results.

C [kW]	I/C [$/kW]	Q_d [MWh]	Q_e [MWh]	k_d []	k_e []
12	3000	28.40	1.82	1.04	0.80
30	2700	52.58	22.95	1.06	0.95
50	2500	62.02	63.86	1.06	0.99
75	2350	68.10	120.74	1.06	1.01
100	2200	71.85	179.91	1.05	1.01

Each of the five linear lines in Figure 1 corresponds to the net present value of one of the five projects from Table 2. An increase in project size results in a steeper net present value function. Even though the smallest project has the highest per kilowatt investment cost it has the lowest net present value break-even. This is because for smaller projects a higher portion of production substitutes own demand at the higher price. However, if the price is higher larger projects have the highest net present value because also exporting electricity is profitable and they have lower investment costs. Someone considering a now or never investment will invest at the upper net present value function if it is over zero, the curve is marked in bold type in Figure 1. It consists of the 12, 30 and 100 kW turbine. The 50 and 75 kW turbines are never optimal. Therefore in the rest of the paper we consider investment in only the three turbines that constitute the upper net preset value curve. Thus $i = 1..3$ corresponds to C equal to 12, 30 and 100 kW.

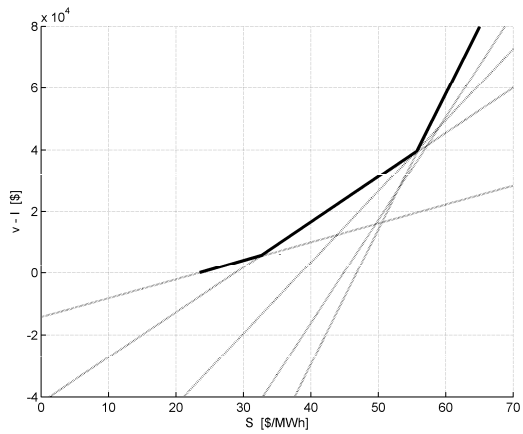

Figure 1: Net present value of five wind turbines with capacity 12, 30, 50, 75 and 100 kW.

Figure 2 shows the solution when the investor has the possibility to postpone the investment. The upper thin line is the value of the investment opportunity. The bold lines are the net present value functions of the investment in the

three different projects. Note that they are no longer linear on the lower branch because they include the option to invest in the larges project when after a project has died. The solution with the base case data is to invest in the 30 kW turbine for values of S between 42.2 and 49.9 $/MWh and to invest in the 100 kW turbine for prices over 61.7 $/MWh. For all other values the optimal decision is to wait for new information about the price. Note that the 12 kW turbine is never optimal because the investment opportunity is always worth more.

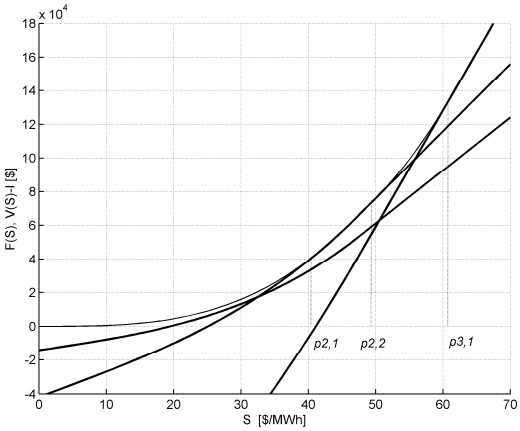

Figure 2: Option to invest in a 12, 30 or a 100 kW turbine.

If there is a high degree of uncertainty about future prices the investment opportunity becomes more valuable. With increased uncertainty there is an increased possibility of both high and low future prices but one will only invest if the price rises. Figure 3 shows the solution with an increased uncertainty parameter of $\sigma = 0.15$. The investment opportunity in the largest project becomes so valuable that investment in the two smaller is never optimal. The optimal strategy is to invest in the 100 kW turbine from $S = 77$ $/MWh.

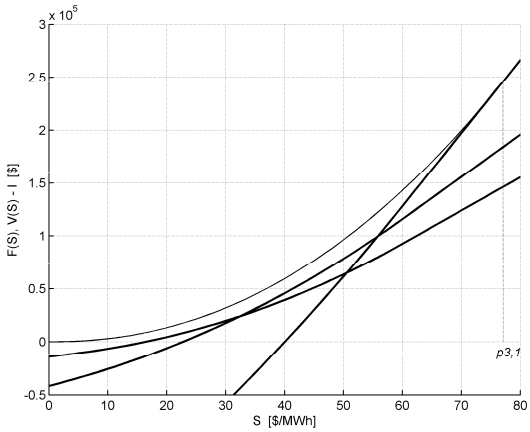

Figure 3: Project values and the value of the investments opportunity in the 12, 30 or 100 kW turbine for $\sigma = 0.15$

Table 3 shows the optimal investment regions for the three projects with changing values of uncertainty. Increasing uncertainty leads to investment at higher prices and in larger projects. Note that when uncertainty goes to zero the investment regions in the 30 kW and 100kW turbine connects. However, increased price growth leads to lower optimal investment prices. At the same time increases the optimal project size. This is because increased growth has lowered the net present value break-even and increased the value of the investment opportunity. The 12 kW turbine is not optimal for any of the values considered. For it to become optimal both price growth and uncertainty must be low such that the decision analysis approaches the Marshallian approach. This indicates that in a price environment with expected price growth and uncertainty the smallest turbine is undersized.

Table 3: Investment regions for the 12, 30 and 100 kW turbines with changing uncertainty and expected price growth.

σ	P_1	P_2	P_3
0	-	35.9-55.8	55.8-
0.025	-	36.9-55.1	55.6-
0.05	-	39.3-53.2	58.8-
0.075	-	42.2-49.9	61.7-
0.1	-	-	65.8-
0.125	-	-	71.6-
0.15	-	-	77.0-
α			
0	-	40.3-55.0	67.2-
1	-	42.2-49.9	61.7-
2	-	-	58.0-
3	-	-	56.4-

DISCUSSION

With the provided example we have shown a method for analysis of investment in small-scale wind power, if the investor can choose both capacity and timing to maximize its benefits. The method results in a recommendation to postpone the investment beyond the net present value break-even price in the market. The optimal investment decision varies with the price. For each capacity that is ever optimal there is a price region where investment is recommended. For the largest capacity the investment threshold is a trigger price where investment is optimal for all higher prices. The results reveal intermediate waiting regions similar to [8] and [9]. This paper does however not assume that the projects has infinite lifetime. Studying a row of investments in perpetuity reduces the intermediate waiting region because the kink in the net present value functions is gentler.

We have assumed that the option to invest is perpetual, which is natural if the investor owns the property with the wind resource. If the investment opportunity has a limited lifetime, the analysis is similar within the lifetime, but when the opportunity expires the decision rule is to choose the capacity that maximizes net present vale and invest in it as long as it is positive.

The method we used is a simplified method. First, we use a relatively simple model of price uncertainty, although it is justifiable for long-term projects. Second, we assume that after a project dies only the option to invest in the largest project is available while in reality the option to invest in any project is available. Therefore the model can

fail to give accurate results if the value of the investment opportunity in the largest project is not important at the price ranges relevant for choosing between two smaller projects. If a preliminary analysis reveals such a situation a smaller project can be used in place of the largest.

The results are based on one example of a customer with only one year of hourly data for consumption and wind speeds. Given these limitations however, the data are general enough to provide some insight into the problem. Further, the price parameters, based on Nord Pool financial data, are only approximate. The decision to postpone the investment is contingent on either an expected growth or uncertainty in the forward price. If there is neither expected growth nor uncertainty, the optimal decision is the static (Marshallian) net present value rule.

We have assumed a constant investment cost over time. To allow for a reduction would complicate the model because of the time dependency but would increase the value of postponing the investment. Further the model does not include tax effects, subsidies or construction time. However, including it is straightforward.

Our analysis optimizes market values and one can argue that small businesses or households are not sufficiently diversified to only care about market value. However, much of the risks can be transferred to the market (e.g. in the form of the local electricity company) by entering into contracts insuring some of the revenue cash flows.

We do not analyze uncertainty in the wind speed because we assume that the wind speed distribution does not vary significantly over the lifetime of the wind mill. Of course, if there are few wind speed measurements available, making it difficult to assess the distribution of wind speed accurately, such measurements are worth paying and/or waiting for. It should also be noted that this uncertainty is a diversifiable risk that can be assumed to have a low market cost to insure. Although there is uncertainty in other parameters, the price uncertainty is most likely to be the dominating uncertain factor. It should however be noted that there is likely uncertainty in the regulation (political risk) and that it can be important but difficult to quantify and incorporate in a model.

Among proponents of distributed generation there is a desire to allow for net metering over a longer period, effectively letting the owner of the windmill receive the higher end-user price for all electricity generated. This would make the investment in distributed generation more worth and save costs for installing hourly metering. An interesting approach for further research is to analyze investments in renewable distributed generation under such regulation and consider investments in either the capacity that exactly matches annual demand or a maximum capacity for the location - sized to produce electricity for exports.

CONCLUSIONS

Motivated by the restructuring in the electricity sector, we have presented a market-based tool for project evaluation under uncertainty for investments in small scale-wind power plants, where the investor has the option to postpone the investment and choose capacity within a range. The optimal investment decision in small-scale wind power depends on a lot of different factors, e.g. electricity load, wind speeds and electricity prices. We have assumed that the future electricity price is the most important factor and have modeled its uncertainty. Our analysis based on data from the Nord Pool financial market with an expected increase in the electricity price and a evident uncertainty in forward prices suggest that the optimal investment decision is to invest at a price considerably over the net present value break-even price. The optimal strategy is to invest in different capacities at different prices ranges. More price volatility and expected price increases make it optimal to wait for higher prices and to invest in larger projects

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